

FINITE CLASSICAL GROUPS

NICK GILL

In this course we study sesquilinear, bilinear and quadratic forms over a vector space, and we study groups of isometries of these forms. Our emphasis will be on vector spaces over finite fields as in this case the set of isometries G is a finite group. This situation is particularly interesting because in ‘most’ cases the group G contains a large simple normal group S and ‘most’ of the finite simple groups crop up in this way. Thus a happy by-product of this course should be that students gain some understanding of the bulk of the finite simple groups.

I envisage that we will cover some subset of the following:

- (1) Important facts about finite groups, particularly permutation groups;
- (2) Some field theory, including possibly a discussion of division rings;
- (3) The linear groups $GL(V)$, $SL(V)$, $PGL(V)$, $PSL(V)$ and the statement of the fundamental theorem of projective geometry;
- (4) Transvections and the simplicity of $PSL_n(q)$;
- (5) Polar spaces, collineations, and polarities;
- (6) Classifying the non-degenerate reflexive forms;
- (7) Isometries, Witt’s Lemma;
- (8) Orthogonal, unitary and symplectic groups;

Acknowledgments and texts. I list relevant texts below. In addition to those listed I have drawn upon notes of Jan Saxl (Cambridge), Tim Penttila (UWA, now Colorado) and Michael Giudici (UWA).

- (1) Cameron, Peter J. *Notes on classical groups*. Lecture notes for a Masters course available for free download at http://www.maths.qmul.ac.uk/~pjc/class_gps/
- (2) Cameron, Peter J. *Projective and polar spaces*. QMW Maths Notes, 13. Queen Mary and Westfield College, School of Mathematical Sciences, London, 199?. An updated version is available for free download at <http://www.maths.qmul.ac.uk/~pjc/pps/>
- (3) Dieudonné, Jean A. *La géométrie des groupes classiques*. Troisième édition. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 5. Springer-Verlag, Berlin-New York, 1971.
- (4) Kleidman, Peter; Liebeck, Martin. *The subgroup structure of the finite classical groups*. London Mathematical Society Lecture Note Series, 129. Cambridge University Press, Cambridge, 1990.
- (5) Taylor, Donald E. *The geometry of the classical groups*. Sigma Series in Pure Mathematics, 9. Heldermann Verlag, Berlin, 1992.
- (6) Wilson, Robert *Finite simple groups*. Lecture notes for an LTCC course available for free download at <http://www.maths.qmul.ac.uk/~raw/FSG/>

Assessment etc. All course material, including lecture notes (posted after the lecture has been given) can be found at the following web page:

<http://users.mct.open.ac.uk/ng3636/fcg.html>

Exercises are scattered through the lectures. Answers will be provided to the starred exercises. (Non-starred are generally either fairly trivial, or else tangential to the direction of the course.)

We will also have an experimental ‘online discussion class’ at 2pm, 16th October (two days after the third lecture). I will give out an extra set of exercises for this, probably in the second lecture, and we will go through them in the class. They will cover material that is connected, but not integral, to the course.

An exam will be set at the end.

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